Day 5 - Review - Rules of Integration AK

Monday, April 22, 2019 9:31 PM

① $\int (x^3 - 3x^2 + x + 4)\,dx$

$y = \frac{1}{4}x^4 - x^3 + \frac{1}{2}x^2 + 4x + C$

② $\int \left(\frac{1}{4}x^{-3} - \sqrt[5]{3}\,x^{2/5}\right)dx$

$y = \frac{-1}{8}x^{-2} - \frac{5\sqrt[5]{3}}{7}x^{7/5} + C$

$y = \frac{-1}{8x^2} - \frac{5\sqrt[5]{3}}{7}x^{7/5} + C$

③ $\int (6x^2+8x)(x^3+2x^2)^4\,dx$

$\int 2(3x^2+4x)\,u^4\,\frac{du}{3x^2+4x}$ $\quad u = x^3+2x^2$ $\quad du = (3x^2+4x)dx$

$\int 2u^4\,du$

$y = \frac{2}{5}u^5 + C$

$y = \frac{2}{5}(x^3+2x^2)^5 + C$

④ $\int \tan(x)\sec^2(x)\,dx$ $\quad u = \sec(x)$

$\int \tan(x)\sec(x)\cdot\sec(x)\,dx$ $\quad du = \sec(x)\tan(x)dx$

$\int \tan(x)\sec(x)\cdot u\,\frac{du}{\sec(x)\tan(x)}$

$\int u\,du$

$y = \frac{1}{2}u^2 + C$

$y = \frac{1}{2}\sec^2(x) + C$

⑤ $\int \frac{1}{\sqrt{x}}\cdot\sin(\sqrt{x})\,dx$ $\quad u = \sqrt{x}$

$u = x^{\frac{1}{2}}$

$\int x^{-1/2}\cdot\sin(u)\,\frac{du}{\frac{1}{2}x^{-1/2}}$ $\quad du = \frac{1}{2}x^{-1/2}dx$

$\int 2\sin(u)\,du = 2\int \sin(u)\,du$

$y = -2\cos(u) + C$

$y = -2\cos(\sqrt{x}) + C$

⑥ $\int \frac{x+2}{\sqrt{x^2+4x+7}}\,dx$ $\quad u = x^2+4x+7$

$du = (2x+4)dx$

$du = 2(x+2)dx$

$\int \frac{x+2}{u^{1/2}}\,\frac{du}{2(x+2)}$

$\frac{1}{2}\int u^{-1/2}\,du$

$y = \frac{1}{2}\cdot 2u^{1/2} + C$

$y = \sqrt{x^2+4x+7} + C$

⑦ $\int (6x^4\cdot\sin^3(4x^5)\cdot\cos(4x^5))\,dx$ $\quad u = \sin(4x^5)$

$\int 6x^4\cdot u^3\cdot\cos(4x^5)\,\frac{du}{20x^4\cdot\cos(4x^5)}$ $\quad du = \cos(4x^5)\cdot 20x^4\,dx$

$\int \frac{6}{20}u^3\,du = \frac{3}{10}\int u^3\,du = \frac{3}{10}\cdot\frac{1}{4}u^4 + C \Rightarrow y = \frac{3}{40}\sin^4(4x^5) + C$

⑧ $\int (3x+2)^2\,dx$

$\int (3x+2)(3x+2)\,dx$

$\int (9x^2+12x+4)\,dx$

$y = 3x^3 + 6x^2 + 4x + C$

⑨ $f'(x) = \int \sqrt[3]{x}\,dx \quad (1,2)$

$y' = \int x^{1/3}\,dx$

$y = \frac{3}{4}x^{4/3} + C$

$2 = \frac{3}{4}(1)^{4/3} + C$

$2 = 3/4 + C$

$5/4 = C$

$\boxed{y = \frac{3}{4}x^{4/3} + 5/4}$

⑩ $f'(x) = \int(\sec^2(x) - \sin(x))\,dx \quad \left(\frac{\pi}{4}, 1\right)$

$y = \tan(x) + \cos(x) + C$

$1 = \tan(\pi/4) + \cos(\pi/4) + C$

⑪ $\frac{dy}{dx} = 5x^{10}y^3 \quad (0,3)$

$\frac{1}{y^3}\,dy = 5x^{10}y^3\,dx$ $\quad \frac{1}{y^3}$ $\quad y^3$

$y = \tan(x) + \cos(x) + C$

$1 = \tan(\pi/4) + \cos(\pi/4) + C$

$1 = 1 + \sqrt{2}/2 + C$

$0 = \sqrt{2}/2 + C$

$-\sqrt{2}/2 = C$

$$\boxed{y = \tan(x) + \cos(x) - \sqrt{2}/2}$$

$$\frac{1}{y^3}\,dy = 5x^{10}\frac{y^3}{y^3}\,dx$$

$$y^{-3}\,dy = 5x^{10}\,dx$$

$$-\frac{1}{2}y^{-2} = \frac{5}{11}x^{11} + C$$

$$\frac{-1}{2y^2} = \frac{5}{11}x^{11} + C$$

$$\frac{-1}{2(3)^2} = \frac{5}{11}(0)^{11} + C$$

$$\frac{-1}{18} = C$$

$$\boxed{\frac{-1}{2y^2} = \frac{5}{11}x^{11} - \frac{1}{18}}$$